THE TRADE-OFF BETWEEN SOME STATE SPACE AND FIR ALGORITHMS IN GPS-BASED OPTIMAL CONTROL OF A LOCAL CRYSTAL CLOCK

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Abstract

The report[1] addresses a numerical investigation of the trade-off between some state space and FIR filtering algorithms intended to provide optimal time error steering of a local crystal clock employing the GPS reference time. For the sake of low-cost crystal clocks, we employ four most simple structures, namely 1) Two-state Kalman, 2) Three-state Kalman, 3) FIR with the constant kernel (simple MA), and 4) FIR with the linear kernel (optimally unbiased for linear phase drift). The optimal control problem is numerically solved in a sense of least mean squares for the GPS-based time error database and for different estimators tuned to have the same time constant. We study a digital control loop of a local clock, assuming the latter to be linear and inertialess with respect to the filter memory. We also compare the algorithms for the mean square error produced and for sensitivity to variations of the feedback coefficient.

INTRODUCTION

Fast and accurate locking of a local clock, along with accurate estimation of its performance (time error), still remains a key problem in GPS-based timekeeping. Usually, it is solved employing two- or three-state Kalman filters via the recursive algorithms; finite impulse response (FIR) filters with a constant, linear, or exponential kernel; and infinite impulse response (IIR) filters with a limited number of poles (as a role, no more than one-two poles). In our previous reports **[1-4]**, it has been numerically shown that no one of the above listed designs is universal for the crystal and rubidium clocks in terms of minimum produced errors. The problem may be distinguished to lie in two planes:

1. ***Uncertainty of a clock time model***, which is postulated by the standards to be of three states (time error, fractional frequency offset, and linear fractional frequency drift rate) and in which all other unnecessary but possibly non-zero states, including noise, are removed to the random phase deviation component;
2. ***Different robustness of filters*** for different frequency offsets and drifts so that the simplest FIR structure with a constant kernel becomes best if the frequency offset falls below some threshold **[4]** and, if not, the problem may be solved in different ways.

One more problem that might also be mentioned is that the clock noise is inherently colored and, thus, the standard Kalman estimator needs to be modified (Kalman claims the noise to be white). Such a modifica-

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tion may be done with additional states. However, in the GPS-based measurement the receiver Gauss noise dominates and, therefore, the Kalman filter may be used straightforwardly.

It has been a well-known rule of thumb that, in contrast to the IIR structure filter, the FIR one is more robust against numerical errors such as computational, quantization, and round-off. Furthermore, since the FIR filter utilizes finite measurements of time errors over the most resent time interval, this filter may demonstrate robustness against temporary uncertainties of the clock time model and even temporary losses of GPS signal. In turn, the IIR structure in such a situation suffers from a divergence phenomenon.

In this report, we numerically investigate the trade-off between the state space Kalman algorithms, which have an IIR structure, and FIR structure algorithms by utilizing them in the clock control loop to solve the filtering and one-step prediction problem. For the sake of low-cost crystal clocks with limited memory, we restrict the studies by employing only four of the most simple filters, such as 1) Two-state (2D) Kalman, 2) Three-state (3D) Kalman, 3) FIR with a constant kernel (CK), and 4) FIR with a linear kernel (LK). We model, simulate, and investigate the GPS-based discrete time control loop of a synchronized time scale, assuming the clock to be linear and inertialess with respect to the filter memory. We find numerically an optimal loop gain for each of the estimators and evaluate the mean square error (MSE) of the steered time scale. We finally bring a numerical example for the GPS-based digital steering of a crystal clock time error, assuming the crystal oscillator to be uncontrollable.

DIGITAL LOOP WITH 1-STEP PREDICTIVE CONTROL OF A LOCAL CLOCK

Figure 1 sketches a GPS-based digital control loop for a local clock. The time error deterministic function of a clock is extended to the Taylor series and traditionally limited to 2-3 states. Accordingly, its vector of 3×1 or 2×1 dimensions reads, respectively, $\boldsymbol{\lambda}_n^T = [x_n y_n D_n]$ or $\boldsymbol{\lambda}_n^T = [x_n y_n]$, where the unknown time error x_n, fractional frequency offset y_n, and linear fractional frequency drift rate D_n are calculated by the recursive equations for the model described by 3-states and 2-states, respectively,

3-states: $$x_n = x_{n-1} + y_{n-1}\Delta + \frac{D_{n-1}}{2}\Delta^2, \ \mathbf{(a)} \quad y_n = y_{n-1} + D_{n-1}\Delta, \ \mathbf{(b)} \quad D_n = D_{n-1}, \ \mathbf{(c)} \qquad (1)$$

2-states: $$x_n = x_{n-1} + y_{n-1}\Delta, \ \mathbf{(a)} \quad y_n = y_{n-1}, \ \mathbf{(b)} \qquad (2)$$

where $\Delta = t_n - t_{n-1}$ is a sample time[1] between two neighboring measurements taken at discrete time instants t_n and t_{n-1}. The random constituent of a clock error suffers of both the oscillator white and colored Gauss noises[2] [e.g., white phase (f^0), flicker phase (f^{-1}), white frequency (f^{-2}), flicker frequency (f^{-3}), and random frequency walk (f^{-4})], so that, generally, the noise vector of 3×1 or 2×1 dimensions reads, respectively, $\boldsymbol{\zeta}_{\lambda n}^T = [\zeta_{xn}\zeta_{yn}\zeta_{Dn}]$ or $\boldsymbol{\zeta}_{\lambda n}^T = [\zeta_{xn}\zeta_{yn}]$, where ζ_{xn}, ζ_{yn}, and ζ_{Dn} are relative zero-mean colored stationary Gauss noises with known covariances $\langle \zeta_{xn}\zeta_{xm} \rangle$, $\langle \zeta_{yn}\zeta_{ym} \rangle$, and $\langle \zeta_{Dn}\zeta_{Dm} \rangle$ or corresponding power spectral densities (psd's). The noises with different psd slopes are usually assumed to be uncorrelated having zero cross-covariances. The deterministic GPS second (1PPS) is supposed to be reference

[1] A sample time is a matter for optimization and for different clocks ranges from tens seconds to 1000 sec [**5**].

[2] Here f^{-l}, where l is integer and f is a Fourier frequency, means the slope of the certain part of the power spectral density (psd) of the oscillator phase.

and, thus, error-free, $x_{GPSn} \equiv 0$. However, at the receiver, the pulses (1PPS) are contaminated by large zero-mean additive white Gauss noise n_{0n} with known (measured) intensity. Accordingly, the reference time error becomes a pure noise. The time interval meter provides measurements of the clock time error employing the GPS reference signal, and this error becomes observable via the noisy vector $\boldsymbol{\xi}_n$.

The estimator first solves a stochastic filtering problem to provide an accurate estimate $\hat{\boldsymbol{\lambda}}_n = \mathscr{F}_n(\xi_{n|n-k})$ of the clock states (1) or (2) via $\boldsymbol{\xi}_n$. In this estimate, the coefficient *k* is equal to unity for the state space model, and it ranges from 1 to *N* for the FIR structure. Respectively, an operator of the estimation $\mathscr{F}_n$ differs. In the holdover case when the GPS timing signals are not temporary available, the estimator forms a recursive 1-step autonomous prediction $\tilde{\boldsymbol{\lambda}}_{n+1} = \mathscr{P}_{n+1}(\hat{\boldsymbol{\lambda}}_{n|n-k+1})$ to make it possible for the algorithm to be robust and for the clock to be locked utilizing the recursive noise-free model. Should the predictor be utilized in the algorithm, the estimation problem is crucial for the clock error steering, and we therefore discuss its solution in detail in the following sections. Finally, the predicted value $\tilde{\boldsymbol{\lambda}}_{n+1}$ is scaled with an unknown negative loop gain $-\mathbf{k}_{n+1}$ and then mixed with the clock 1-step ahead actual value $\boldsymbol{\lambda}_{n+1}$. The optimal control law is then a linear feedback law that specifies $\boldsymbol{\lambda}_{n+1}$ to be $\boldsymbol{\lambda}_{n+1} = -\mathbf{k}_{n+1}\tilde{\boldsymbol{\lambda}}_{n+1}$, where the feedback coefficients matrix for 3-states and 2-states is described by, respectively,

$$\mathbf{k}_n = \begin{bmatrix} k_{xn} & 0 & 0 \\ 0 & k_{yn} & 0 \\ 0 & 0 & k_{Dn} \end{bmatrix} \text{ and } \mathbf{k}_n = \begin{bmatrix} k_{xn} & 0 \\ 0 & k_{yn} \end{bmatrix}, \tag{3}$$

in which the components k_{xn}, k_{yn}, and k_{Dn} are intended to govern each of the clock states.

To obtain a time-invariant value of the feedback gain (3) over the observable database of *M* points, the MSE $\langle \varepsilon_{n+1}^2 \rangle = E[(\boldsymbol{\lambda}_{n+1} + \mathbf{k}_{n+1}\tilde{\boldsymbol{\lambda}}_{n+1})^2]$ is minimized for the optimal feedback gain $\mathbf{k}_0$. The optimization problem then appears to minimize the quadratic performance index (cost function)

$$\langle \varepsilon^2 \rangle_{\min} = E\{[\boldsymbol{\lambda}_{n+1} + \mathbf{k}_0 \mathscr{P}_{n+1}(\mathscr{F}_n(\xi_{n|n-k}))]^2\}_{\substack{k \le n \le k+M-1 \\ 1 \le k \le N}} \tag{4a}$$

$$= \frac{1}{M}\left\langle \sum_{k}^{k+M-1} (\boldsymbol{\lambda}_{n+1} + \mathbf{k}_0 \tilde{\boldsymbol{\lambda}}_{n+1})^2 \right\rangle, \ \forall k = 1,\ldots,N. \tag{4b}$$

Such a problem may be solved in two ways, namely by employing either the state space (IIR) model ($k = 1$) or the FIR structure ($k = N$). Below we consider both cases.

STATE SPACE PREDICTIVE ESTIMATION OF TIME ERROR

In the state space, the clock state equation and the observation equation are given by, respectively,

$$\boldsymbol{\lambda}_n = \mathbf{A}\boldsymbol{\lambda}_{n-1} + \mathbf{u}_n + \boldsymbol{\zeta}_{\lambda n}\,, \ (\mathbf{a}) \quad \xi_n = \mathbf{H}\boldsymbol{\lambda}_n + n_{0n}\,, \ (\mathbf{b}), \tag{5}$$

where $\mathbf{u}_n^T = [u_{1n}u_{2n}...u_{mn}]$ is vector of a control signal of $m\times 1$ dimensions (for the clock model, $m = 2$ or 3) and dimensions of all other vectors and matrixes depend on the number of the states. The noises $\boldsymbol{\zeta}_{\lambda n}$ and n_{0n} are jointly uncorrelated, so that for all k and n there is $E(n_{0n}\boldsymbol{\zeta}_{\lambda k}) = 0$. The GPS discrete stationary white Gauss noise n_{0n} has zero mean, $\langle n_{0n} \rangle = 0$, with known covariance

$$E(n_{0n}n_{0k}) = \begin{cases} V_n = \sigma_{0n}^2, & k = n \\ 0, & k \neq n \end{cases}, \tag{6}$$

which is usually assumed to be time-invariant, $\sigma_{0n}^2 = V = \sigma_0^2 = N_0 / 2\Delta$, where $N_0 / 2$ is a double-sided psd of the corresponding continues noise. In the clock control problem, we ignore the fact that the noise $\boldsymbol{\zeta}_{\lambda n}$ is colored and assume it to be white[3]. Accordingly, we go to the idealized covariance matrix

$$E(\boldsymbol{\zeta}_{\lambda n}\boldsymbol{\zeta}_{\lambda k}^T) \cong E(\mathbf{n}_{\lambda n}\mathbf{n}_{\lambda k}^T) = \begin{cases} \boldsymbol{\Psi}, & k = n \\ 0, & k \neq n \end{cases}, \tag{7}$$

in which dimensions of $\boldsymbol{\Psi}$ depend on the number of the states. For the sake of definiteness, we finish now with a consideration of two aforementioned cases of the state space filter structures.

Case I—Two-state filter structure. *When an oscillator noisy phase is assumed to be Brownian with a linear drift, only two states become non-zero and, thus, the state transition matrix of 2×2 dimensions, measurement matrix of 2×1 dimensions, and error matrix of 2×2 dimensions are formed by, respectively,*

$$\mathbf{A} = \begin{bmatrix} 1 & \Delta \\ 0 & 1 \end{bmatrix}, (\mathbf{a}), \ \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}, (\mathbf{b}), \ \boldsymbol{\Psi} \cong \frac{N_y\Delta}{2}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, (\mathbf{c}), \tag{8}$$

where $N_y / 2$ *is a double-sided uniform psd of the second state noise, which is dependent on a sample time* Δ*. We notice that the quantity* $N_y\Delta^2 / N_0$ *represents the desired (or conventionally assumed) signal-to-noise ratio (SNR) in the observation* ξ_n.

Case II—Three-state filter structure. *Assuming the oscillator phase to be Brownian with a quadratic drift leads to the 3-state situation, in which the state transition matrix of 3×3 dimensions, measurement matrix of 3×1 dimensions, and error matrix of 3×3 dimensions become, respectively,*

[3] In such a model, the noise is not oscillator's white phase (f^0) but rather a permitted time error for the locked clock

$$\mathbf{A}=\begin{bmatrix}1 & \Delta & \Delta^2/2\\ 0 & 1 & \Delta\\ 0 & 0 & 1\end{bmatrix}, \textbf{(a)},\ \mathbf{H}=\begin{bmatrix}1 & 0 & 0\end{bmatrix}, \textbf{(b)},\ \mathbf{\Psi}\cong\frac{N_D\Delta}{2}\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}, \textbf{(c)}, \quad (9)$$

where $N_D/2$ is a double-sided uniform psd of the third state noise, which depends on Δ.. Like the case (8), here the quantity $N_D\Delta^2/N_0$ represents the desired SNR in the observation ξ_n.

The linear Kalman estimation algorithm associated with the filtering and 1-step prediction problem reads

Enter $\mathbf{R}_{n-1}$ and $\hat{\boldsymbol{\lambda}}_{n-1}$

Prediction (project ahead): $$\tilde{\mathbf{R}}_n = \mathbf{A}\mathbf{R}_{n-1}\mathbf{A}^T + \mathbf{\Psi}, \quad (10)$$

Compute **Kalman gain**: $$\mathbf{K}_n = \tilde{\mathbf{R}}_n\mathbf{H}^T(\mathbf{H}\tilde{\mathbf{R}}_n\mathbf{H}^T + \mathbf{V})^{-1}, \quad (11)$$

Update **estimate** with measurement ξ_n: $$\hat{\boldsymbol{\lambda}}_n = \mathbf{A}\hat{\boldsymbol{\lambda}}_{n-1} + \mathbf{K}_n(\boldsymbol{\xi}_n - \mathbf{u}_n - \mathbf{H}\mathbf{A}\hat{\boldsymbol{\lambda}}_{n-1}), \quad (12)$$

Compute **error covariance** for updated estimate: $$\mathbf{R}_n = (\mathbf{I} - \mathbf{K}_n\mathbf{H})\tilde{\mathbf{R}}_n, \quad (13)$$

Make 1-step ahead, $n = n + 1$, and go to (10).

Compute **1-step prediction**: $$\tilde{\boldsymbol{\lambda}}_{n+1|n} = \mathbf{A}\hat{\boldsymbol{\lambda}}_{n|n}, \quad (14)$$

Predict **1-step ahead error covariance**: $$\tilde{\mathbf{R}}_{n+1|n} = \mathbf{A}\mathbf{R}_{n|n}\mathbf{A}^T + \mathbf{\Psi}, \quad (15)$$

where $\hat{\boldsymbol{\lambda}}_{n|n}$ in (14) is the estimate (12) taken for the observed n points and $\mathbf{R}_{n|n}$ in (15) is the error covariance associated with the estimate $\hat{\boldsymbol{\lambda}}_{n|n}$.

PREDICTIVE FIR FILTER FOR TIME ERROR ESTIMATION

For FIR structures, the clock time error equations (1) and (2) modify to

3-states: $$x_n = x_0 + y_0\Delta n + \frac{D}{2}\Delta^2 n^2, \textbf{(a)},\quad y_n = y_0 + D\Delta n, \textbf{(b)},\quad D_n = D, \textbf{(c)}, \quad (16)$$

2-states: $$x_n = x_0 + y_0\Delta n, \textbf{(a)},\quad y_n = y_0, \textbf{(b)}, \quad (17)$$

where x_0, y_0, and D are initial constant values. The time error model and its observation equation are then

$$\boldsymbol{\lambda}_n = \mathbf{A}_n\boldsymbol{\lambda}_0 + \mathbf{u}_n + \boldsymbol{\zeta}_{\lambda n}, \textbf{(a)},\quad \xi_n = \mathbf{H}\boldsymbol{\lambda}_n + n_{0n}, \textbf{(b)}, \quad (18)$$

where for the constant initial state vectors $\boldsymbol{\lambda}_0^T = [x_0 y_0 D]$ and $\boldsymbol{\lambda}_0^T = [x_0 y_0]$ the transition matrixes of 3×3 and 2×2 dimensions become nonstationary, respectively,

$$\mathbf{A}_n = \begin{bmatrix} 1 & \Delta n & \Delta^2 n^2/2 \\ 0 & 1 & \Delta n \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{A}_n = \begin{bmatrix} 1 & \Delta n \\ 0 & 1 \end{bmatrix}. \tag{19}$$

In view of (18b) and $\mathbf{H}\boldsymbol{\lambda}_n = x_n$, the FIR filtering estimate of a time error reads

$$\hat{x}_n = \sum_{i=0}^{N-1} W_i \xi_{n-i} = \mathbf{z}_N^T \mathbf{W}_N = \mathbf{x}_N^T \mathbf{W}_N + \mathbf{n}_N^T \mathbf{W}_N, \tag{20}$$

where W_i is a weighting function (kernel or impulse response) of a FIR filter; $\mathbf{z}_N^T(n) = [\xi_n \xi_{n-1} ... \xi_{n-N+1}]$ is the measurement data vector, of dimensions N×1; $\mathbf{W}_N^T = [W_0(N), W_1(N), ..., W_{N-1}(N)]$ is the filter weight matrix, of dimensions N×1; $\mathbf{x}_N^T(n) = [x_n x_{n-1} ... x_{n-N+1}]$ is the time error data vector, of dimensions N×1, and $\mathbf{n}_N^T(n) = [n_n n_{n-1} ... n_{n-N+1}]$ is the GPS noise data vector, of dimensions N×1. Inherently, the first estimate provided by (20) appears at the point n with a delay on the transient

$$\theta = \Delta(N-1). \tag{21}$$

So long as in the control strategy a predictor is removed out of the FIR filter, a 1-step linear prediction is formed for (20) in the recursive form

$$x_{n+1|n} = \hat{x}_{n|n} + E(\hat{x}_{n-k|n-k} - \hat{x}_{n-k-1|n-k-1}), \tag{22}$$

where an average increment of the time error estimates is taken for the reasonable number k of measurements in the nearest past. Let us notice that there may be utilized some other predictive algorithms based, for example, on the regression approximation **[1]** and polynomial FIR predictor structure **[6]**. The mean square error (MSE) of the FIR estimate $\varepsilon_{xn} = x_n - \hat{x}_n$ over the observed database is calculated by $\langle \varepsilon_x^2 \rangle = E(\varepsilon_x^2) = \Delta\hat{x}_n^2 + \sigma_{xn}^2$, where the estimate bias and variance are computed by, respectively,

$$\Delta\hat{x}_n = E(x_n - \mathbf{x}_N^T \mathbf{W}_N - \mathbf{n}_N^T \mathbf{W}_N), \text{ (a)}, \quad \sigma_{xn}^2 = E[(x_n - \mathbf{x}_N^T \mathbf{W}_N - \mathbf{n}_N^T \mathbf{W}_N - \Delta\hat{x}_n^2)^2], \text{ (b)}. \tag{23}$$

We notice that, in contrast to the state space estimator whose structure is exhaustedly determined by the number of the states and properly filled matrixes, the waiting function of the FIR filter is a matter for optimization. Below we consider two cases suitable for the GPS-based locking of crystal clocks.

Case III—FIR structure with a constant kernel. *The CK FIR filter is also known as a simple moving average (MA) filter that is optimal in a sense of minimum produced noise. Its weight is rectangular, i.e.* $\mathbf{W}_N = N^{-1}\mathbf{I}_N$, *where* $\mathbf{I}_N = [1 \ 1 \ ...1]^T$ *is a unit matrix of dimensions N×1. A disadvantage is a large bias when a time error function demonstrates an easily seen nonstationarity. Errors associated with the filter application to the Brownian phase with a linear drift are examined in* ***[4]***.

Case IV—FIR structure with a linear kernel*. The LK FIR filter is optimally unbiased when a time error function behaves as a Brownian phase with a linear drift. The filter was designed in* **[7]** *with its weighting function*

$$W_i = \begin{cases} \dfrac{2(2N-1)-6i}{N(N+1)}, & 0 \le i \le N-1 \\ 0, & \text{otherwise} \end{cases}. \tag{24}$$

Statistical errors of this filter for the Brownian phase with a linear drift are discussed in **[4]***. An advantage is that, having a relatively simple kernel (24), the filter demonstrates an intermediate performance between the three-state and two-state Kalman filters and this is independent of the time error function.*

NUMERICAL STUDIES OF OPTIMAL LOOP GAIN

We continue on with numerical studies of the loop (Figure 1) employing four above-described estimators. To compare the results, we exploit the same measurement of the time error of a crystal clock. We use the Motorola GPS UT+ Oncore Timing receiver; the low-cost crystal oscillator, AT-cut, 5 MHz in the local clock; and the time interval meter with a resolution of 1 ns. A sample time is set to be 100s. In parallel, we measure the time error for the reference rubidium standard. We numerically solve the control problem (4b) for the steered time error (1-state), assuming the oscillator to be uncontrollable (indirect steering). A typical result is shown in Figure 2, where ξ_n is a GPS-based measurement; λ_n is an actual time error measured for the rubidium standard; $\xi_n + u_n$ is a controlled measurement; and $\lambda_n + u_n$ is a time error of a locked clock. To evaluate the trade-off, we set the same time constant for all filters. In doing so for the Kalman filters, which has a IIF structure, we set the noise psd $N_y/2$ in (8c) or $N_D/2$ in (9c) to finish the transient of the solution of the discrete time Riccati equation (10) at the level of 0.95 with respect to the first state. We then solve the control problem (4b) for the optimal feedback gain k_{0x}.

Figure 3 shows the RMSE of the locked clock as function of k_{xn} for several numbers N of the points in the average, employing the 2D Kalman (a) and 3D Kalman (b). Figure 4 makes it for the CK FIR (a) and LK FIR (b). In these figures, the minimum value of k_{xn} is optimal in a sense of LMS. For the sake of processing accuracy, we refrain from setting $N > 150$, having a limited database. We notice that, by increasing k_{xn}, the clock loop evolves from a large error to its minimum, and then goes through the intermittence to divergence (full instability). The generalization is shown in Figure 5. Here, the first plot (a) shows how the optimal gain k_{0x} changes if N increases, and the second one (b) demonstrates the same for the RMSE. It is neatly seen that the unbiased filters (2D and 3D Kalman, and LK FIR) behave with quite some similarity, whereas the CK FIR, with its large bias, demonstrates an opposite sign of the slopes.

DISCUSSION

GPS-based locking of a local clock assumes estimation of the time error with its following steering in the closed loop. Very often, the estimation is provided by employing FIR and state space structures. To determine the trade-off between these filters, we simulated and studied a digital control loop of a local clock for four most simple filters: 2D Kalman, 3D Kalman, CK FIR, and LK FIR. Setting the same time constant for all filters, exploiting the only typical measurement obtained for the crystal clock, and assuming the oscillator of the clock to be uncontrollable (direct discrete control of a time scale), we arrive at the following conclusions:

- With small *N*, when the noise dominates in the averaging, the CK FIR produces better result over all other examined estimators;
- With large *N*, the 2D Kalman is best and the LK FIR occupies an intermediate place between the 3D and 2D Kalman;
- The 2D Kalman and CK FIR have, respectively, less and more sensitive structures to variations in the optimal feedback gain.

Overall, we notice that, for the sake of accuracy, the estimator needs to be designed to possess a structure that guarantees minimum error for all feasible situations with the clock time error. Such an estimator may be produced in different ways **[8]**. However, important constrains are: 1) the algorithm must be simple (it should not be burdensome for clock engineers), 2) it must be robust, and 3) it must rely on small memory of a low-cost crystal clock.

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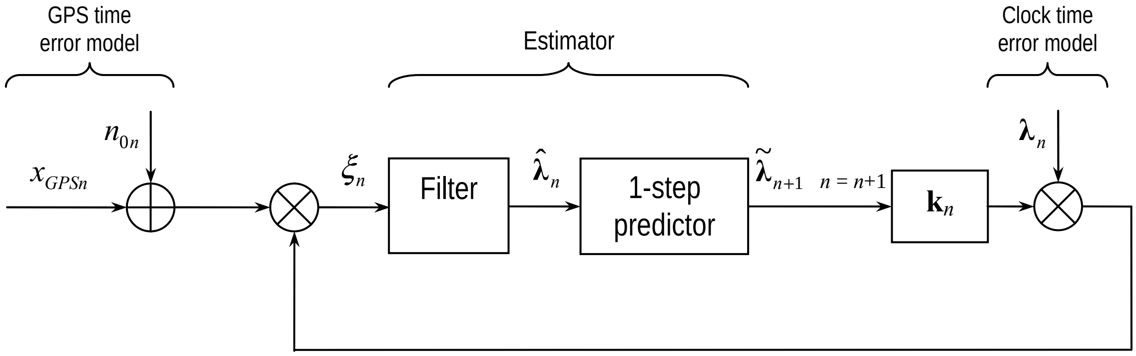


Figure 1. A discrete time model of GPS-based steering of a local clock time error with an indirect control.

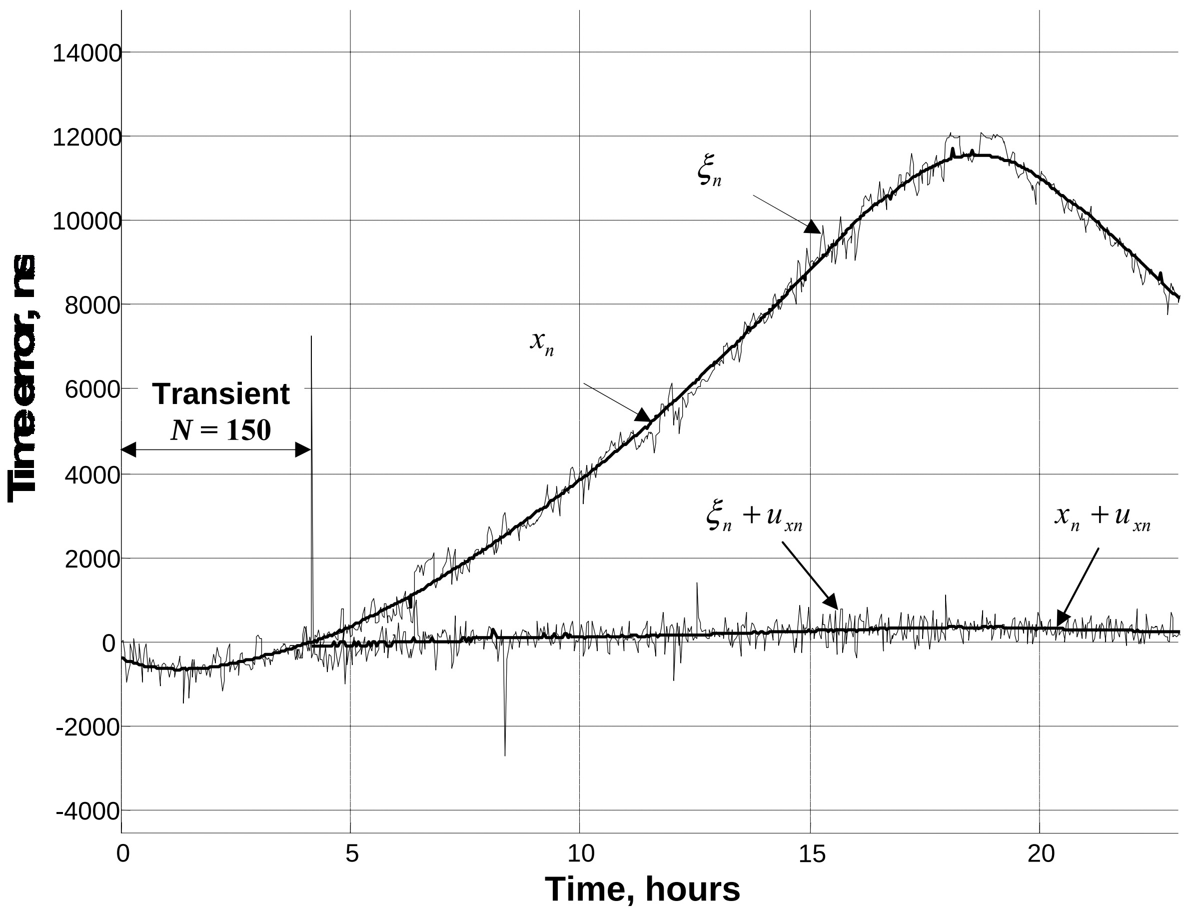


Figure 2. Typical measurement of a time error of a crystal clock: ξ_n is a GPS-based observation, x_n is an actual time error measured for the rubidium reference clock, $\xi_n + u_{xn}$ means an observation in the control loop, and $x_n + u_{xn}$ means a steered time error.

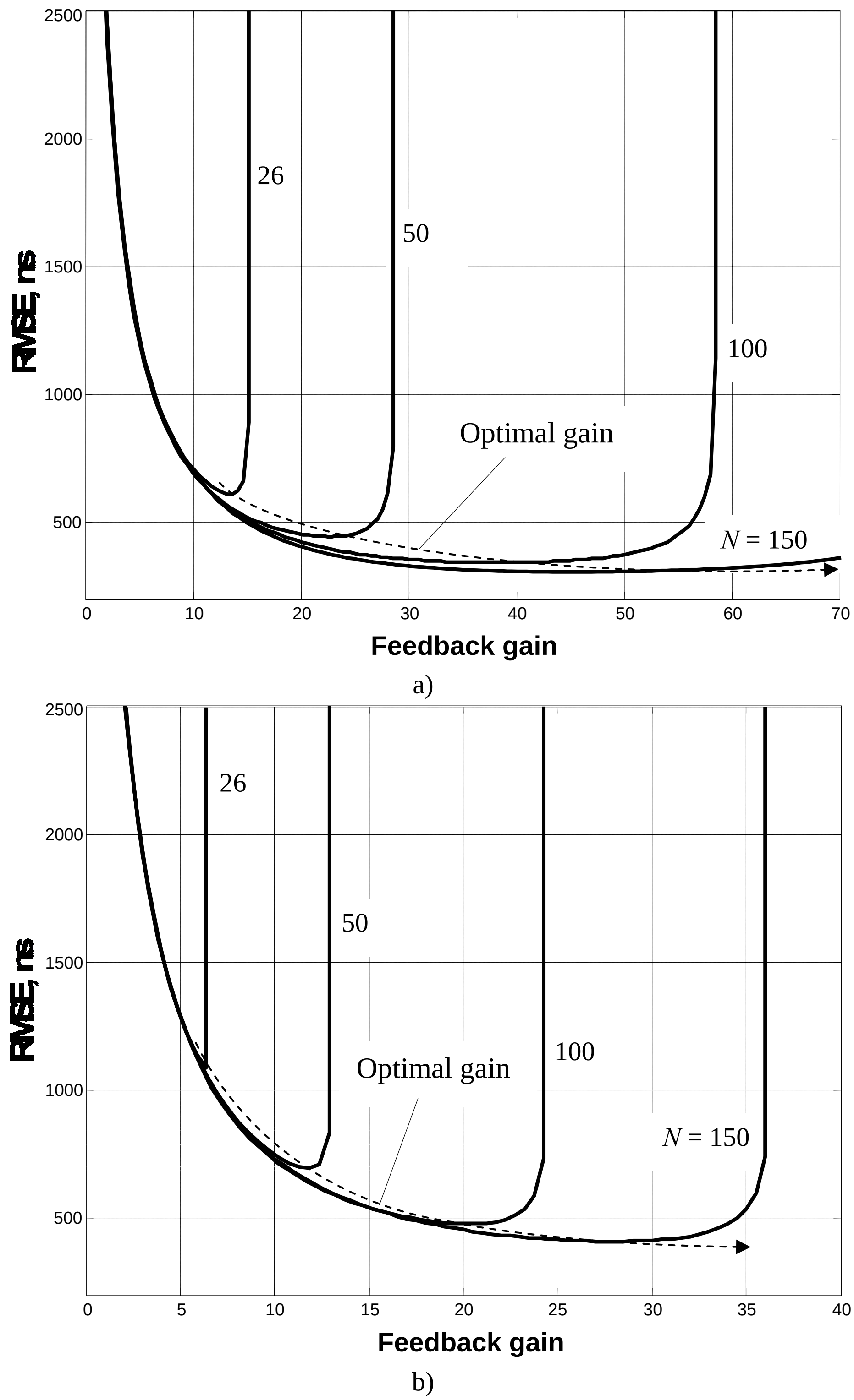
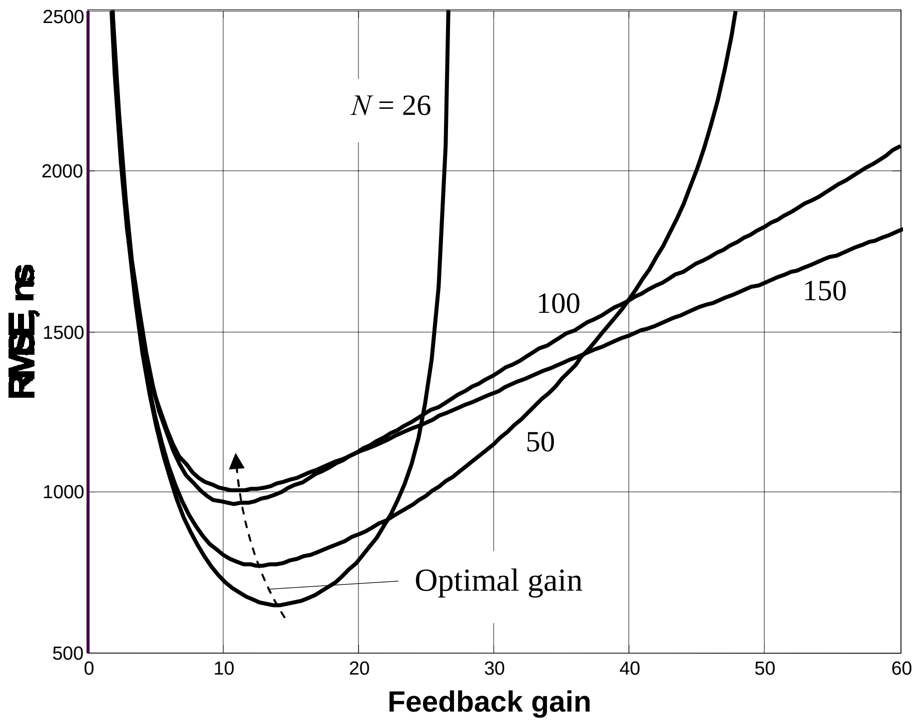
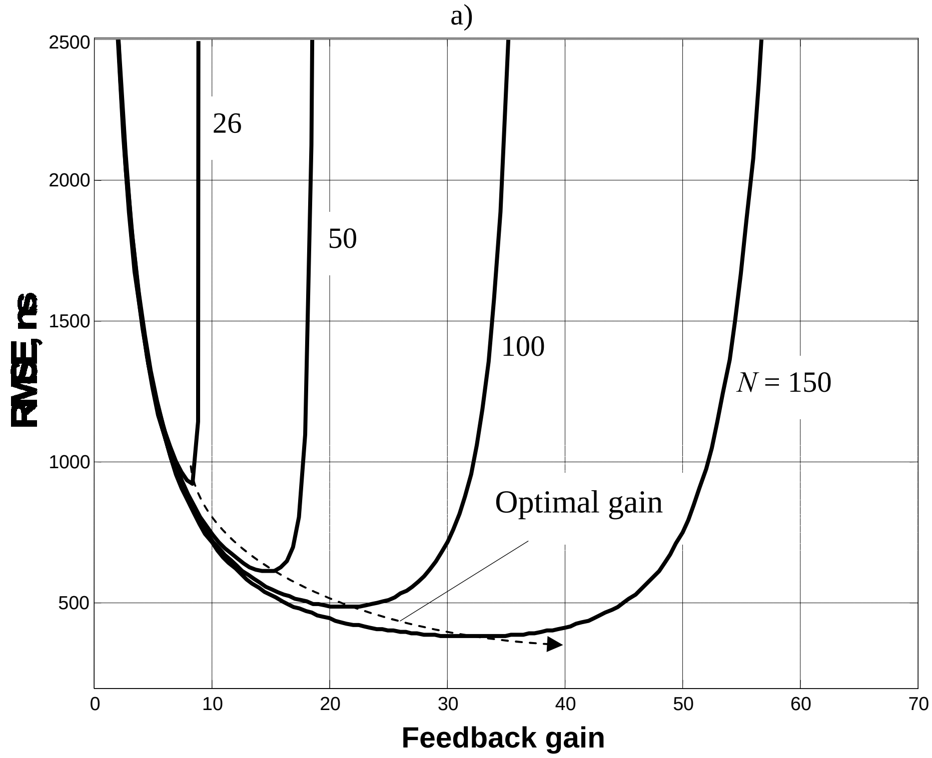


Figure 3. RMSE of the steered crystal clock as function of the feedback gain for several numbers N of the points in the filter transient: a) 2D Kalman, b) 3D Kalman.

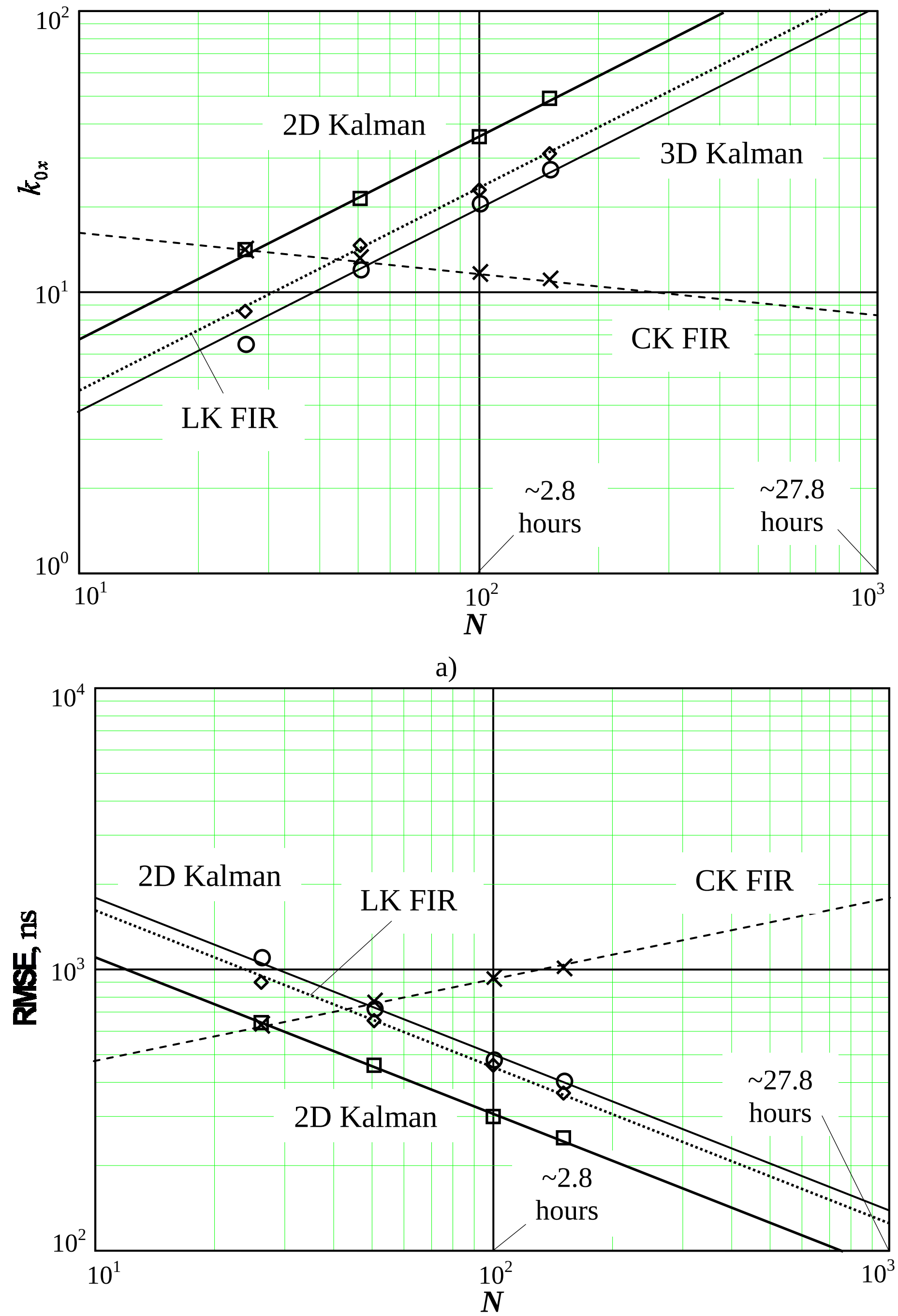


a)



b)

Figure 4. RMSE of the steered crystal clock as function of the feedback gain for several numbers *N* of the points in the filter kernel: a) CK FIR, b) LK FIR.



a)

b)

Figure 5. Statistical performance of the steered crystal clock for four filters as function of a number N of the points in the average: a) Optimal feedback coefficient k_{0x}, b) RMSE.